Defects

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Plan:

What is a defect

Integrable defects, transmission equation (RTT) nogo theorem

Folding to a boundary theory

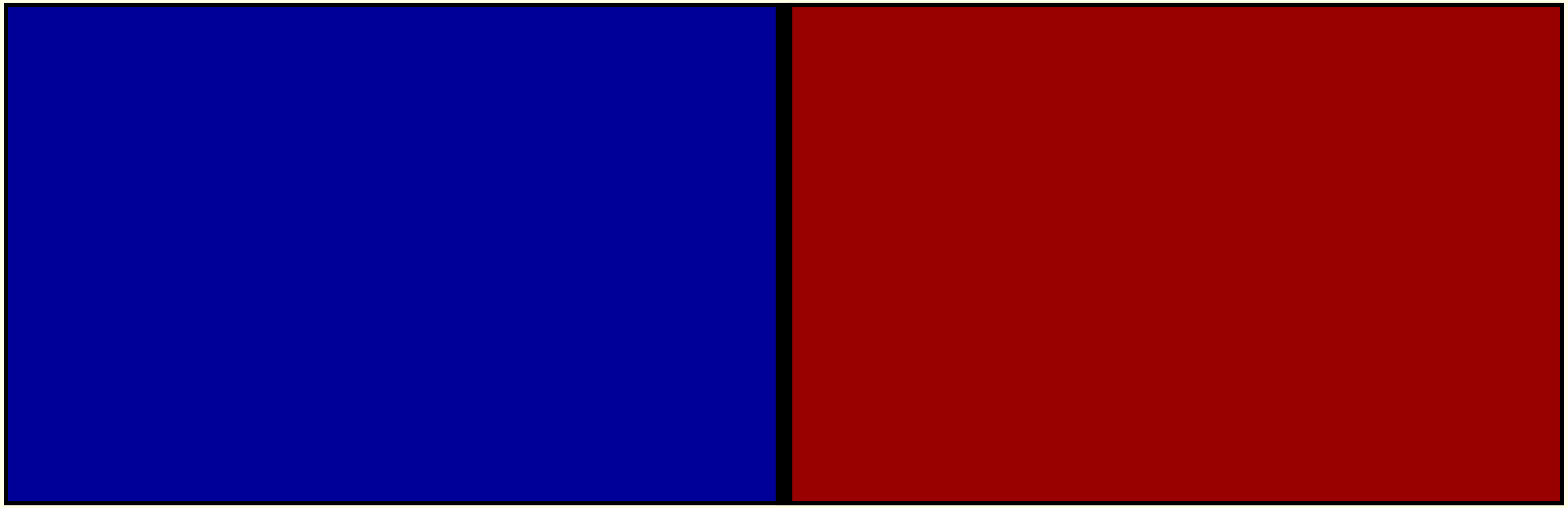
Purely transmitting defects

Defects and boundaries

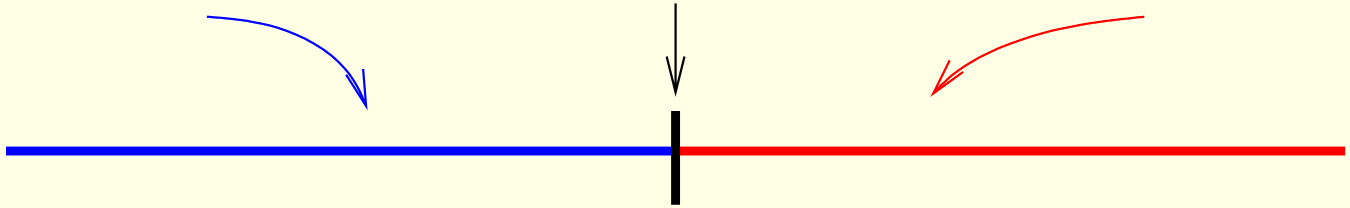
Defects in the sine-Gordon theory and XXZ

Derivation of the constraint of Rafael

What is a defect?



$$\tfrac{1}{2}(\partial_t\Phi)^2-\tfrac{1}{2}(\partial_x\Phi)^2-V(\Phi) \qquad U(\Phi,\dot{\Phi},\Psi,\dot{\Psi}) \qquad \tfrac{1}{2}(\partial_t\Psi)^2-\tfrac{1}{2}(\partial_x\Psi)^2-W(\Psi)$$



Relativistic bosonic field in 1+1 dimensions

Integrable aspects

Assumptions:

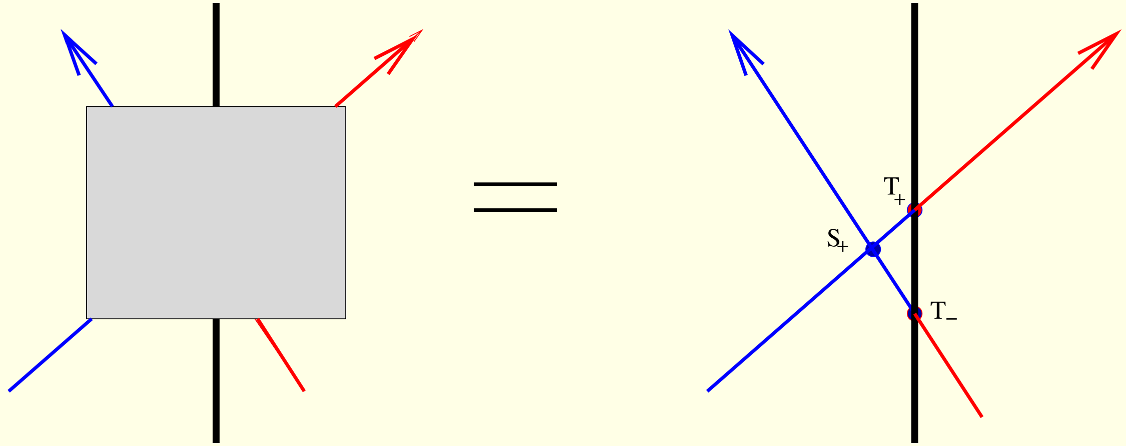
1. Factorization:

into the products of the

bulk S-matrices $S_\pm$

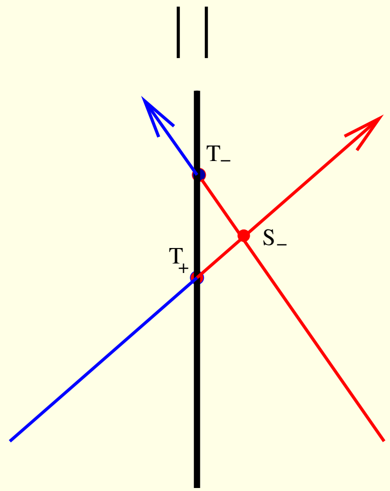
and reflections

and transmissions



2. Shifting of the trajectories:

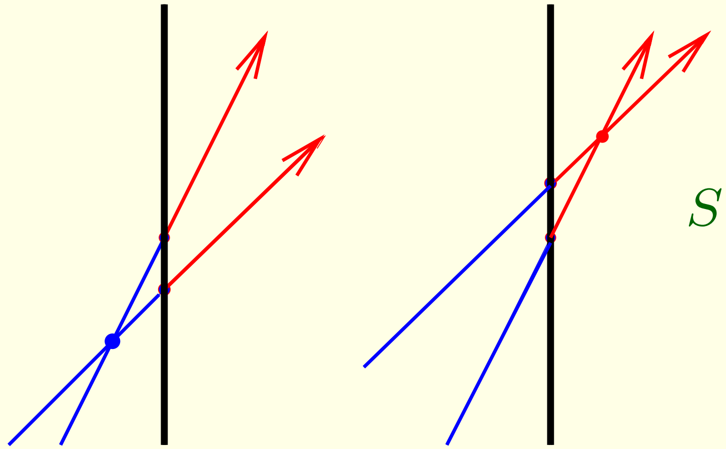
Yang-Baxter equation



Transmission equations
$T_- S_+ T_+ = T_+ S_- T_-$

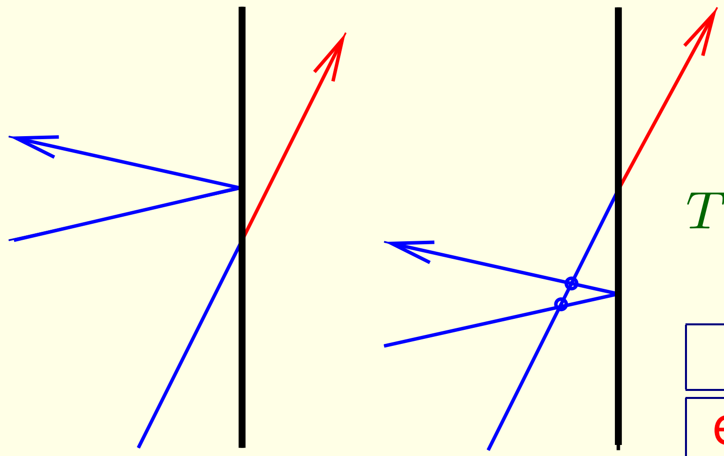
Shifting the defect

Transmission equation, No-go theorem



$$S(\theta_1 - \theta_2)T(\theta_1)T(\theta_2) = T(\theta_2)T(\theta_1)S(\theta_1 - \theta_2)$$

RTT relation



$$T(\theta_1)R(\theta_2) = S(\theta_1 - \theta_2)R(\theta_2)S(\theta_1 + \theta_2)T(\theta_1)$$

No-go theorem:
either no simultaneous reflection and transmission
or the system is free

History of nonfree theories

1. Additional assumptions leading to no-go :

Diagonal bulk S-matrix:

G. Delfino, G. Mussardo and P. Simonetti, *Phys.Lett. B* **328** (1994)

G. Delfino, G. Mussardo and P. Simonetti, *Nucl. Phys. B* **432** (1994) 518

Non-diagonal bulk S-matrix, generic dynamical boundary:

O.A. Castro-Alvaredo, A. Fring and F. Göhmann hep-th/0201142.

2. Assumptions avoiding the no-go:

S-matrix is not the bulk S-matrix

M. Mintchev, E. Ragoucy, P. Sorba, *Phys.Lett. B* **547** (2002) 313
M. Mintchev, E. Ragoucy, P. Sorba, *J.Phys. A* **36** (2003) 10407
V. Caudrelier, M. Mintchev, E. Ragoucy:*J.Phys. A***37** (2004) L367-L376 ....

3. Purely transmitting defects

R. Konik and A. LeClair, *Nucl.Phys. B***538** (1999) 587-611

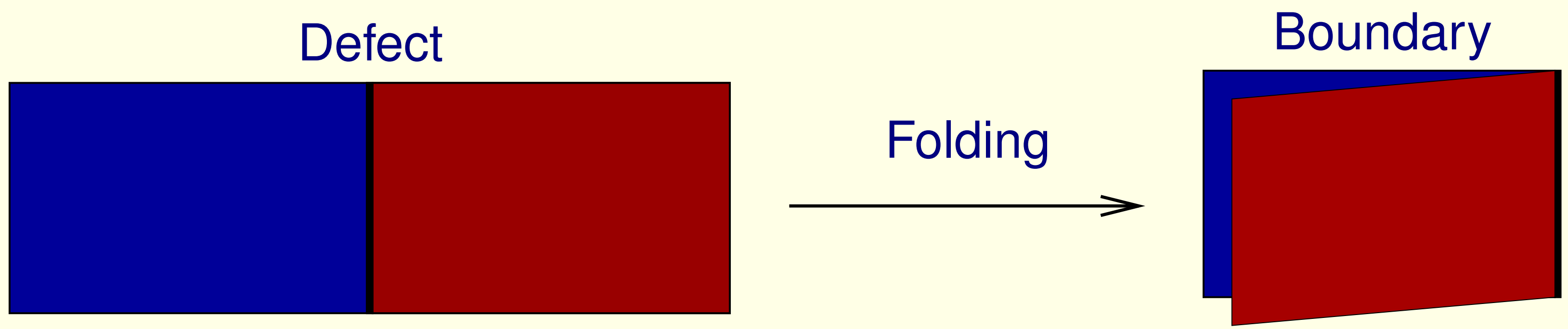
P. Bowcock, E. Corrigan, C. Zambon,*Int.J.Mod.Phys. A***19S2** (2004) 82-91

P. Bowcock, E. Corrigan, C. Zambon, *JHEP* **0401** (2004) 056

P. Bowcock, E. Corrigan, C. Zambon, hep-th/0506169

Folding

Basic idea:



Consequence: results for defects $\longleftarrow$ results from boundaries

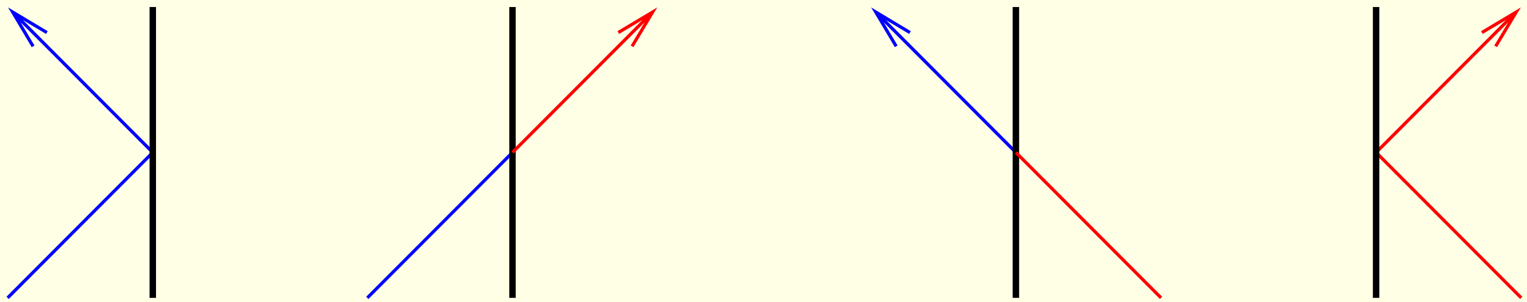
Folding history:

Conformal theories
J. Erdmenger, Z. Guralnik and I. Kirsch, *Phys.Rev. D* **66** (2002) 025020
T. Quella and V. Schomerus, *JHEP* **0206** (2002) 028.
Free theories
M. Oshikawa and I. Affleck, *Nucl.Phys. B* **495** (1997) 533.
H. Saleur, Lectures on Non Perturbative Field Theory and Quantum Impurity Problems, *preprint:* cond-mat/9812110.

RT and $\mathcal{R}$ matrix

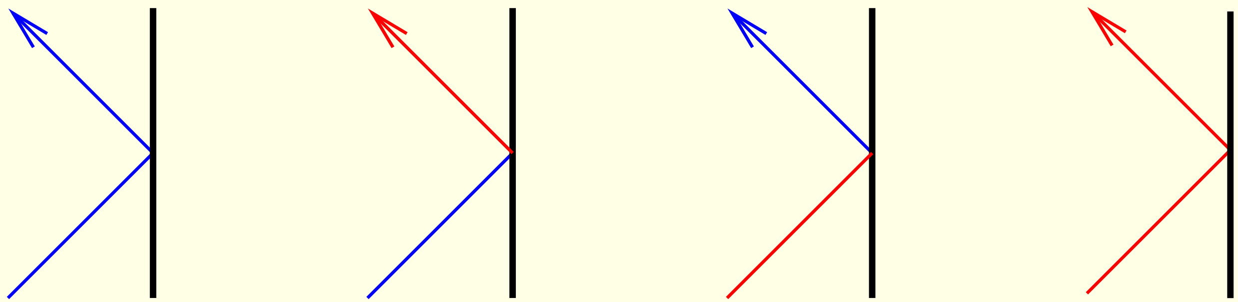
Asymptotic states are connected by RT and $\mathcal{R}$ matrix

Defect RT matrix



R_+ T_+ T_- R_-

Boundary $\mathcal{R}$ matrix



$\mathcal{R}_{\Phi\Phi}$ $\mathcal{R}_{\Phi\Psi}$ $\mathcal{R}_{\Psi\Phi}$ $\mathcal{R}_{\Psi\Psi}$

Integrable aspects:

Assumptions: for both the defect and the boundary systems

1. Existence of infinite independent higher spin charges:

Trajectories can be shifted $\longrightarrow$ Yang-Baxter equations

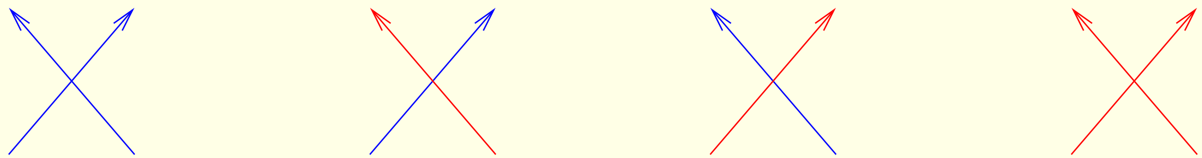
2. Factorization into the product of:

Defect   Bulk S-matrix: $S_\pm$ Reflections: $R_+\,,\ R_-$ Transmissions: $T_+\,,\ T_-$

Boundary   Bulk S-matrix: $\mathcal{S}$ Reflections: $\mathcal{R}_{\Phi\Phi}, \mathcal{R}_{\Psi\Psi}$ Reflections: $\mathcal{R}_{\Phi\Psi}, \mathcal{R}_{\Psi\Phi}$

How are $S_\pm$ and $\mathcal{S}$ related to each other?

From the Lagrangian via the boundary reduction formula:

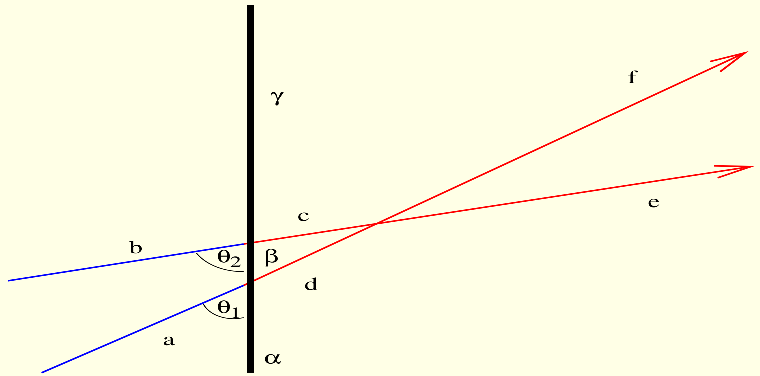


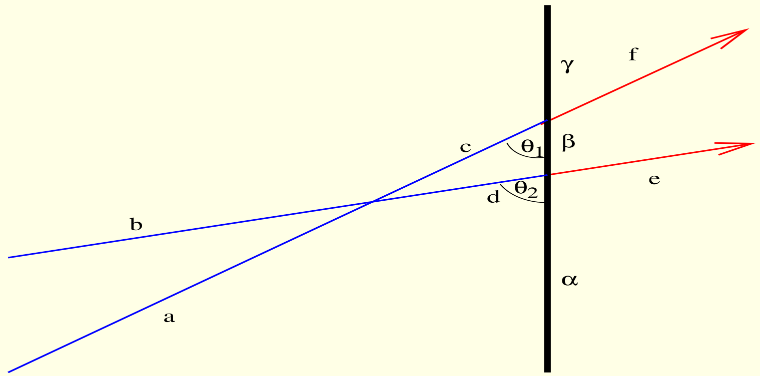
$\mathcal{S}_{\Phi\Phi} = S_+$ $\mathcal{S}_{\Phi\Psi} = 1$ $\mathcal{S}_{\Psi\Phi} = 1$ $\mathcal{S}_{\Psi\Psi} = S_-$

Now the YB for the defect and for the boundary systems are EQUIVALENT

Example: transmission

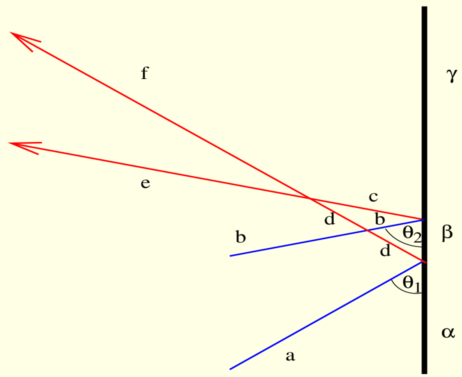
Yang Baxter equation in the defect case

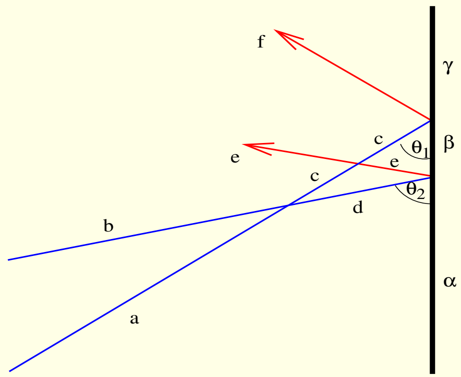




$$T_{+}{}^{\beta d}_{\alpha a}(\theta_1)\,T_{+}{}^{\gamma c}_{\beta b}(\theta_2)\,S_{-}{}^{fe}_{dc}(\theta_2-\theta_1) \quad = \quad S_{+}{}^{cd}_{ab}(\theta_2-\theta_1)\,T_{+}{}^{\beta e}_{\alpha d}(\theta_2)\,T_{+}{}^{\gamma f}_{\beta c}(\theta_1)\ .$$

Yang Baxter equation in the boundary case





$$\mathcal{R}_{\Phi\Psi}{}^{\beta d}_{\alpha a}(\theta_1)\,\mathcal{S}_{\Phi\Psi}{}^{bd}_{bd}(\theta_1+\theta_2)\,\mathcal{R}_{\Phi\Psi}{}^{\gamma c}_{\beta b}(\theta_2)\,\mathcal{S}_{\Psi\Psi}{}^{fe}_{dc}(\theta_2-\theta_1)$$
$$= \mathcal{S}_{\Phi\Phi}{}^{cd}_{ab}(\theta_2-\theta_1)\,\mathcal{R}_{\Phi\Psi}{}^{\beta e}_{\alpha d}(\theta_2)\,\mathcal{S}_{\Psi\Phi}{}^{ce}_{ce}(\theta_1+\theta_2)\,\mathcal{R}_{\Phi\Psi}{}^{\gamma f}_{\beta c}(\theta_1)$$

Boundary results $\to$ Defect results

1. Boundary unitarity

$$\mathcal{R}_{a\alpha}^{b\beta}(\theta)\mathcal{R}_{b\beta}^{c\gamma}(-\theta)=\delta_a^c\delta_\alpha^\gamma$$

Defect unitarity

$$\begin{aligned}
R_{+}{}_{\alpha a}^{\beta b}(\theta)R_{+}{}_{\beta b}^{\gamma c}(-\theta)+T_{+}{}_{\alpha a}^{\beta b}(\theta)T_{-}{}_{\beta b}^{\gamma c}(-\theta) &= \delta_a^c\delta_\alpha^\gamma \\
T_{-}{}_{a\alpha}^{b\beta}(\theta)T_{+}{}_{b\beta}^{c\gamma}(-\theta)+R_{-}{}_{a\alpha}^{b\beta}(\theta)R_{-}{}_{b\beta}^{c\gamma}(-\theta) &= \delta_a^c\delta_\alpha^\gamma \\
R_{+}{}_{\alpha a}^{\beta b}(\theta)T_{+}{}_{b\beta}^{c\gamma}(-\theta)+T_{+}{}_{\alpha a}^{\beta b}(\theta)R_{-}{}_{\beta b}^{\gamma c}(-\theta) &= 0 \\
T_{-}{}_{\alpha a}^{\beta b}(\theta)R_{+}{}_{\beta b}^{\gamma c}(-\theta)+R_{-}{}_{\alpha a}^{\beta b}(\theta)T_{-}{}_{\beta b}^{\gamma c}(-\theta) &= 0
\end{aligned}$$

2. Boundary crossing unitarity (BCU)

$$\mathcal{R}_{\alpha a}^{\beta b}(\theta)=\mathcal{S}_{ad}^{cb}(2\theta)\mathcal{R}_{\alpha\bar{d}}^{\beta\bar{c}}(i\pi-\theta)$$

Defect crossing unitarity

$$R_{+}{}_{\alpha a}^{\beta b}(\theta)=S_{+}{}_{ad}^{cb}(2\theta)R_{+}{}_{\alpha\bar{d}}^{\beta\bar{c}}(i\pi-\theta)\ , \quad ; \quad T_{+}{}_{\alpha a}^{\beta b}(\theta)=T_{-}{}_{\alpha\bar{b}}^{\beta\bar{a}}(i\pi-\theta)\ ,$$
$$R_{-}{}_{\alpha a}^{\beta b}(\theta)=S_{-}{}_{ad}^{cb}(2\theta)R_{-}{}_{\alpha\bar{d}}^{\beta\bar{c}}(i\pi-\theta)\ , \quad ; \quad T_{-}{}_{\alpha a}^{\beta b}(\theta)=T_{+}{}_{\alpha\bar{b}}^{\beta\bar{a}}(i\pi-\theta)\ ,$$

Boundary results $\longrightarrow$ Defect results

3. Boundary state

$$
\begin{aligned}
|B\rangle \; = \; \exp\Bigg\{ \int_0^\infty \frac{d\theta}{2\pi} \Bigg(& R_{\Phi\Phi}\left(\frac{i\pi}{2}-\theta\right) A^+_\Phi(-\theta) A^+_\Phi(\theta) + R_{\Phi\Psi}\left(\frac{i\pi}{2}-\theta\right) A^+_\Phi(-\theta) A^+_\Psi(\theta) \\
& + R_{\Psi\Phi}\left(\frac{i\pi}{2}-\theta\right) A^+_\Psi(-\theta) A^+_\Phi(\theta) \Bigg) + R_{\Psi\Psi}\left(\frac{i\pi}{2}-\theta\right) A^+_\Psi(-\theta) A^+_\Psi(\theta) \Bigg\}
\end{aligned}
$$

Defect operator

$$
\begin{aligned}
D \; = \; \exp\Bigg\{ \int_0^\infty \frac{d\theta}{2\pi} \Bigg(& R_{+}\left(\frac{i\pi}{2}-\theta\right) A^+_\Phi(-\theta) A^+_\Phi(\theta) + T_{+}\left(\frac{i\pi}{2}-\theta\right) A^+_\Phi(-\theta) A_\Psi(-\theta) \\
& + T_{-}\left(\frac{i\pi}{2}-\theta\right) A_\Psi(\theta) A^+_\Phi(\theta) \Bigg) + R_{-}\left(\frac{i\pi}{2}-\theta\right) A_\Psi(\theta) A_\Psi(-\theta) \Bigg\}
\end{aligned}
$$

4. Boundary bootstrap method

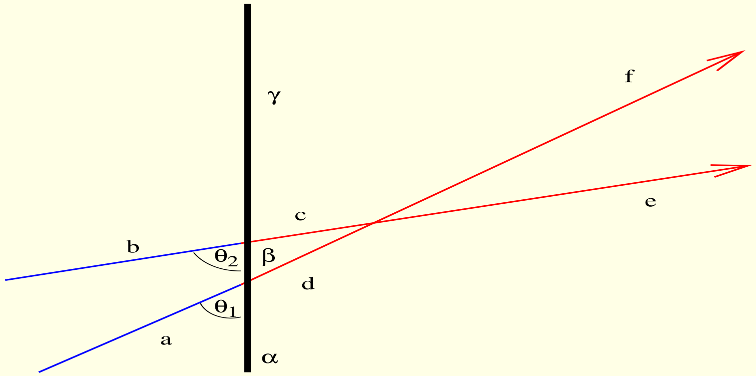
Given a solution satisfying	BYBE Unitarity BCU	Bootstrap $\longrightarrow$	New solution satisfying	BYBE Unitarity BCU

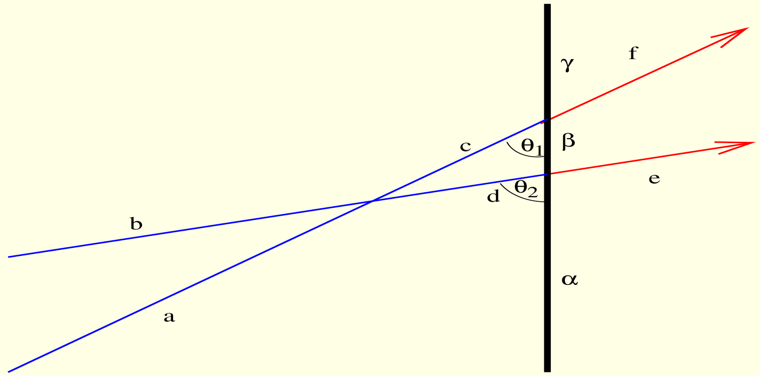
Defect bootstrap method to generate new solutions

DEFECTS are FREE (Casimir) or PURELY TRANSMITTING

Purely transmitting defects

Transmission equation





$$T_{+\,\alpha a}^{\;\beta d}(\theta_1)\,T_{+\,\beta b}^{\;\gamma c}(\theta_2)\,S_{-\,dc}^{\;fe}(\theta_2-\theta_1) \quad = \quad S_{+\,ab}^{\;cd}(\theta_2-\theta_1)\,T_{+\,\alpha d}^{\;\beta e}(\theta_2)\,T_{+\,\beta c}^{\;\gamma f}(\theta_1)\ .$$

Unitarity

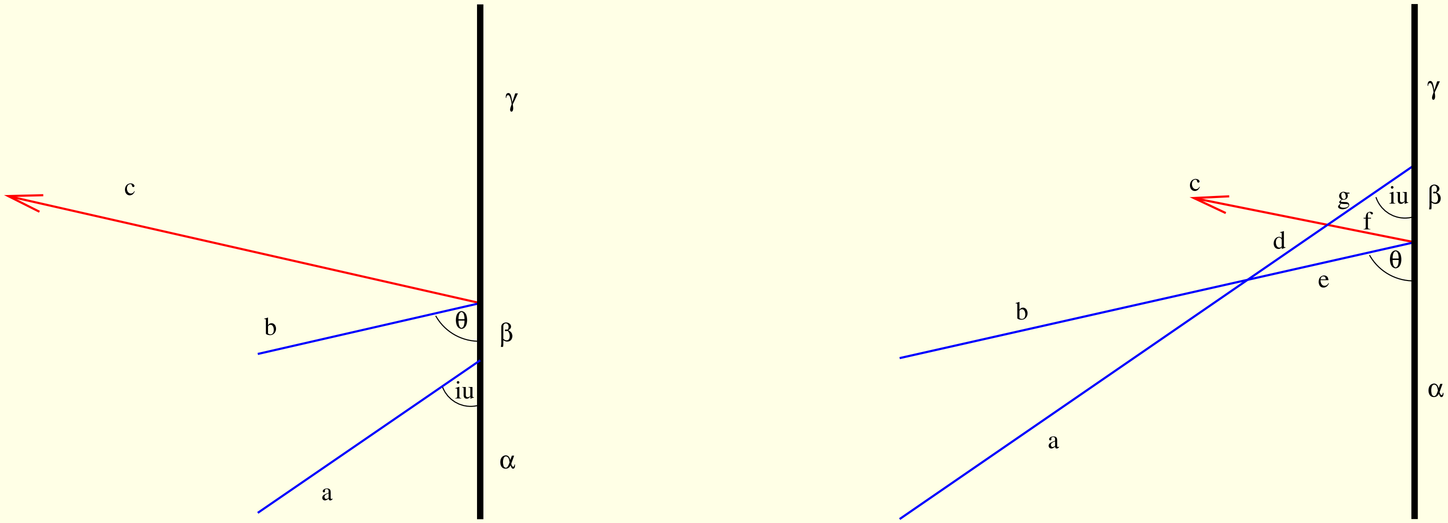
$$T_{+\,\alpha a}^{\;\beta b}(\theta)\,T_{-\,\beta b}^{\;\gamma c}(-\theta) = \delta_a^c \delta_\alpha^\gamma \quad ; \quad T_{-\,a\alpha}^{\;b\beta}(\theta)\,T_{+\,b\beta}^{\;c\gamma}(-\theta) = \delta_a^c \delta_\alpha^\gamma$$

Crossing unitarity

$$T_{+\,\alpha a}^{\;\beta b}(\theta) = T_{-\,\alpha\bar{b}}^{\;\beta\bar{a}}(i\pi-\theta)\ , \quad ; \qquad T_{-\,\alpha a}^{\;\beta b}(\theta) = T_{+\,\alpha\bar{b}}^{\;\beta\bar{a}}(i\pi-\theta)\ ,$$

Example: purely transmitting defect with $S_- = S_+$

1. Boundary bootstrap



$$g^{\beta}_{a\alpha}\mathcal{R}^{c\gamma}_{b\beta}(\theta) \qquad = \qquad g^{\gamma}_{g\beta}\mathcal{S}^{de}_{ab}(\theta - iu)\mathcal{R}^{f\beta}_{e\alpha}(\theta)\mathcal{S}^{eg}_{fd}(\theta + iu)$$

Trivial solution: $T_+ = 1\ ;\ T_- = 1$

Defect bootstrap method

Non-trivial solution:

$$T_{+}{}^{cd}_{ba}(\theta) = S_{+}{}^{cd}_{ab}(\theta - iu)\ ; \quad T_{-}{}^{cd}_{ba}(\theta) = S_{-}{}^{dc}_{ba}(\theta + iu)\ .$$

"Defect behaves like a standing" particle

Purely transmitting defects and boundaries, dressing

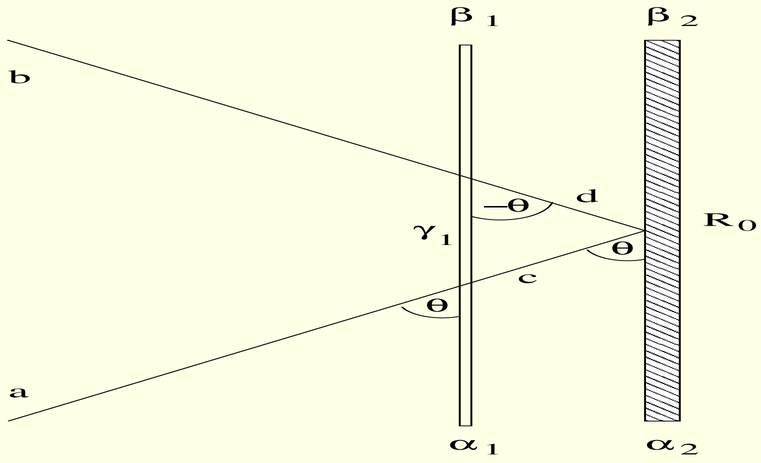
Dressing boundaries (Sklyanin):

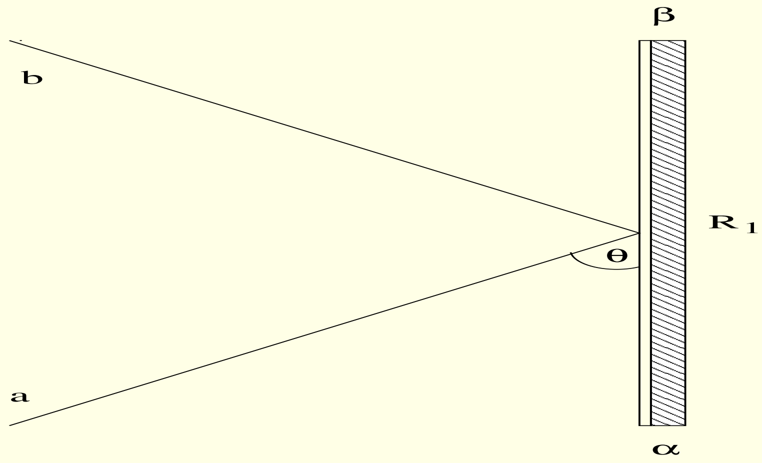
$T_{+}{}^{b\beta}_{a\alpha}(\theta)$, $T_{-}{}^{b\beta}_{a\alpha}(\theta)$, $R_0{}^{b\beta}_{a\alpha}(\theta)$ $\longrightarrow$ $R_1{}^{b\beta}_{a\alpha}(\theta)$

Defect Boundary Boundary

1. YBE 2. Unitarity 3. Crossing unitarity

$$T_{+}{}^{c\gamma_1}_{a\alpha_1}(\theta)R_0{}^{d\beta_2}_{c\alpha_2}(\theta)T_{-}{}^{b\beta_1}_{d\gamma_1}(\theta) = R_1{}^{b\beta}_{a\alpha}(\theta)$$



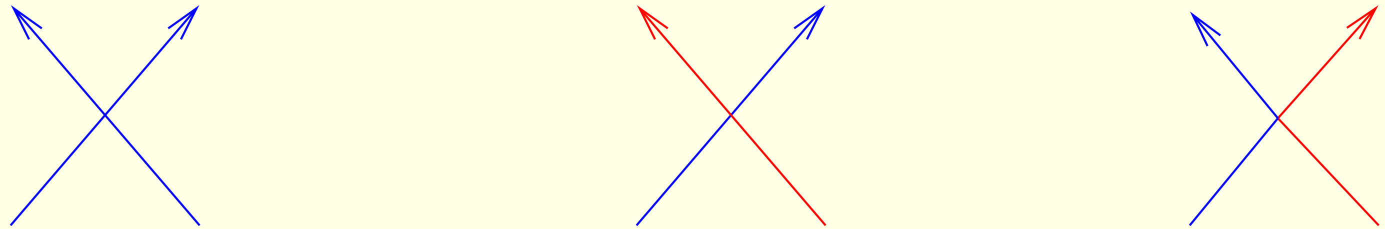


Generate solutions of BYBE from transmission equation

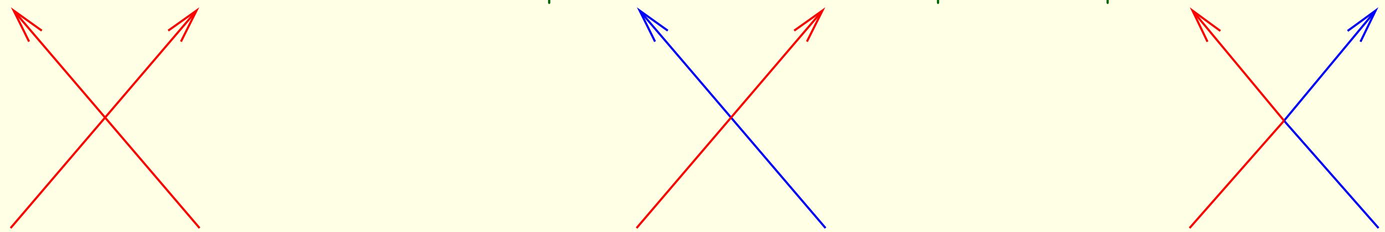
Example: Sine-Gordon model, Bulk

Bulk particle spectrum: soliton (+) and antisoliton (-) with S-matrix

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{-\sin i\lambda\theta}{\sin\lambda(\pi+i\theta)} & \frac{\sin\lambda\pi}{\sin\lambda(\pi+i\theta)} & 0 \\ 0 & \frac{\sin\lambda\pi}{\sin\lambda(\pi+i\theta)} & \frac{-\sin i\lambda\theta}{\sin\lambda(\pi+i\theta)} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \prod_{l=1}^{\infty}\left[\frac{\Gamma(2(l-1)\lambda+\frac{\lambda i\theta}{\pi})\Gamma(2l\lambda+1+\frac{\lambda i\theta}{\pi})}{\Gamma((2l-1)\lambda+\frac{\lambda i\theta}{\pi})\Gamma((2l-1)\lambda+1+\frac{\lambda i\theta}{\pi})}/(\theta\to-\theta)\right]$$



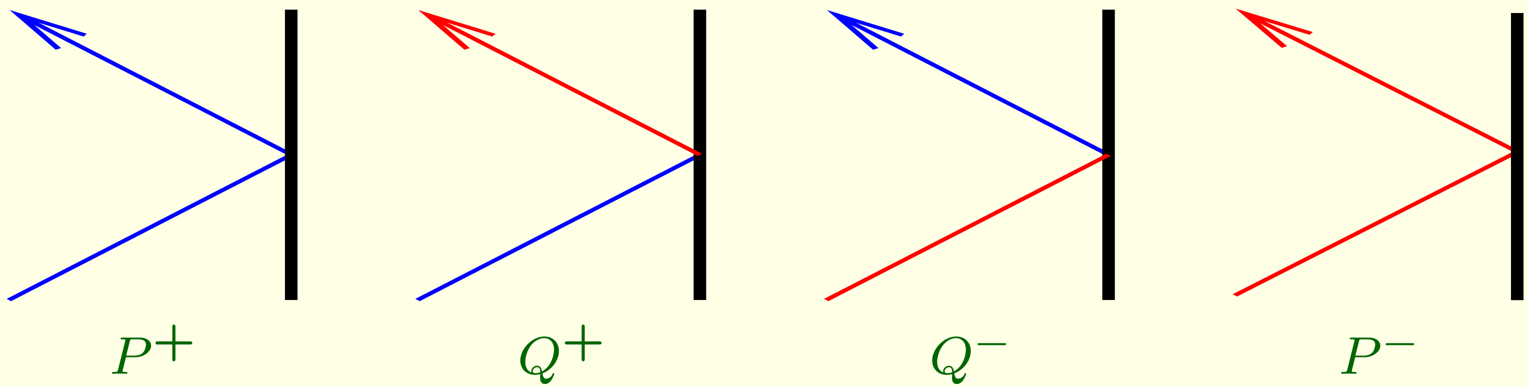
$S_{++}^{++} = \sin\lambda(\pi+i\theta) = S_{--}^{--}$ $\qquad$ $S_{+-}^{+-} = \sin i\lambda\theta = S_{-+}^{-+}$ $\qquad$ $S_{+-}^{-+} = \sin\lambda\pi = S_{-+}^{+-}$



Example: Sine-Gordon model, Boundary

Most general reflection factors satisfying boundary Yang-Baxter equation, unitarity, boundary crossing symmetry.

$$R(\eta,\Theta,\gamma)=\begin{pmatrix} P^{+} & Q^{+} \\ Q^{-} & P^{-} \end{pmatrix} R_0(\theta)\frac{\sigma(\eta,\theta)}{\cos\eta}\frac{\sigma(i\Theta,\theta)}{\cosh\Theta}$$



$$P^{\pm}=\cos(i\lambda\theta)\cos\eta\cosh\Theta\pm(\cos\leftrightarrow\sin)\quad;\quad Q^{\pm}=e^{\pm i\gamma}\cos i\lambda\theta\sin i\lambda\theta$$

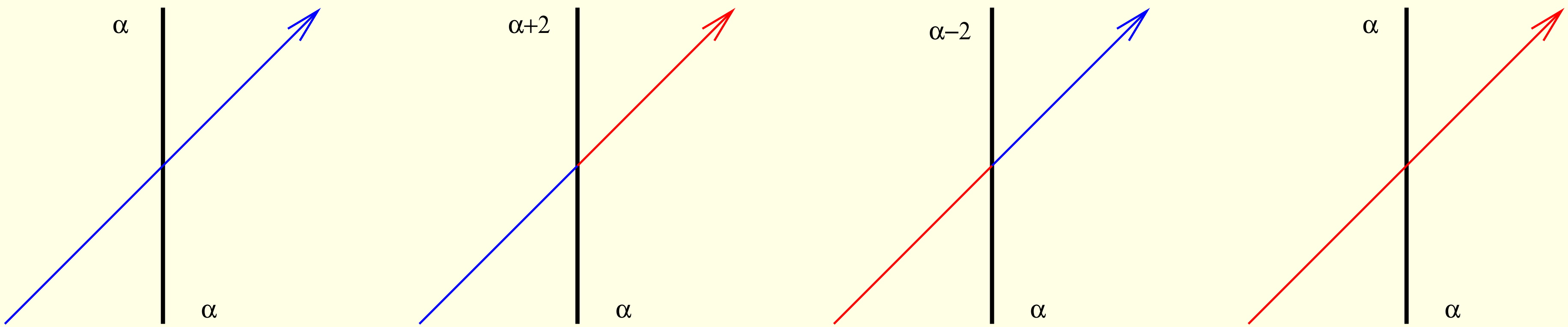
Diagonal solution: $\Theta\to\infty$ Dirichlet $R_D(\eta)$

$$Q^{\pm}\to 0\quad;\quad P^{\pm}\to\cos(\eta\mp i\lambda\theta)\quad;\qquad \sigma\to 1$$

Example: Sine-Gordon model, Defect

Most general reflection factors satisfying boundary Yang-Baxter equation, unitarity, boundary crossing symmetry.

$$T_{\alpha\beta}(\mu,\xi,\zeta) = \begin{pmatrix} q^{\alpha/2}P^{+} & q^{-\alpha/2}Q^{+} \\ q^{-\alpha/2}Q^{-} & q^{\alpha/2}P^{-} \end{pmatrix}_{\alpha\beta} T_0(\mu,\theta)$$



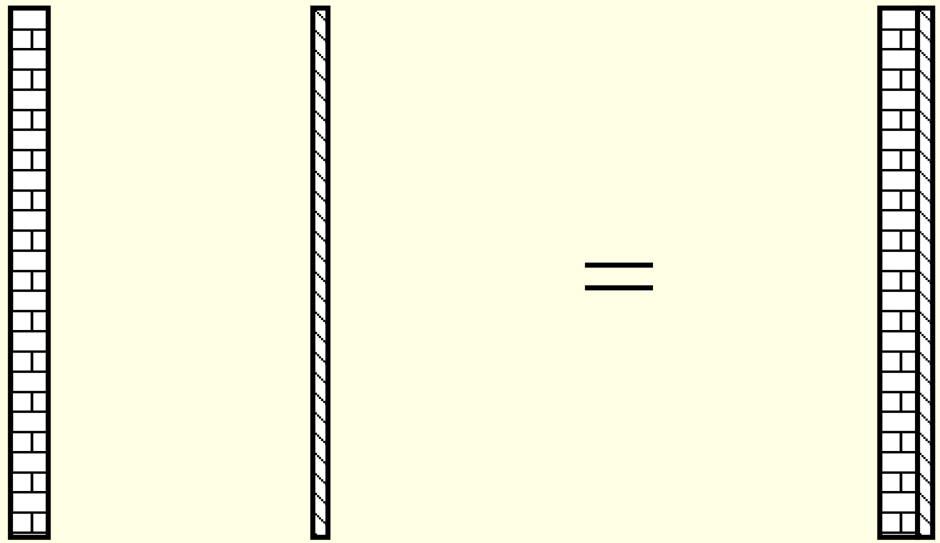
$q^{\alpha/2}P^{+}$ $\quad$ $q^{\alpha/2}Q^{+}$ $\quad$ $q^{-\alpha/2}Q^{-}$ $\quad$ $q^{-\alpha/2}P^{-}$

$$P^{\pm} = e^{\pm i(\xi+\zeta)/2} e^{-(\lambda\theta+\mu)} \quad ; \quad Q^{\pm} = \pm e^{\pm i(\xi-\zeta)/2} \quad ; \quad q = -e^{i\pi\lambda}$$

Example: sine-Gordon theory, dressing

Dressing boundaries:

$R_D(\eta) \qquad T(\mu,\xi,\zeta) \qquad\qquad\qquad R(\eta,\mu,\xi+\eta)$

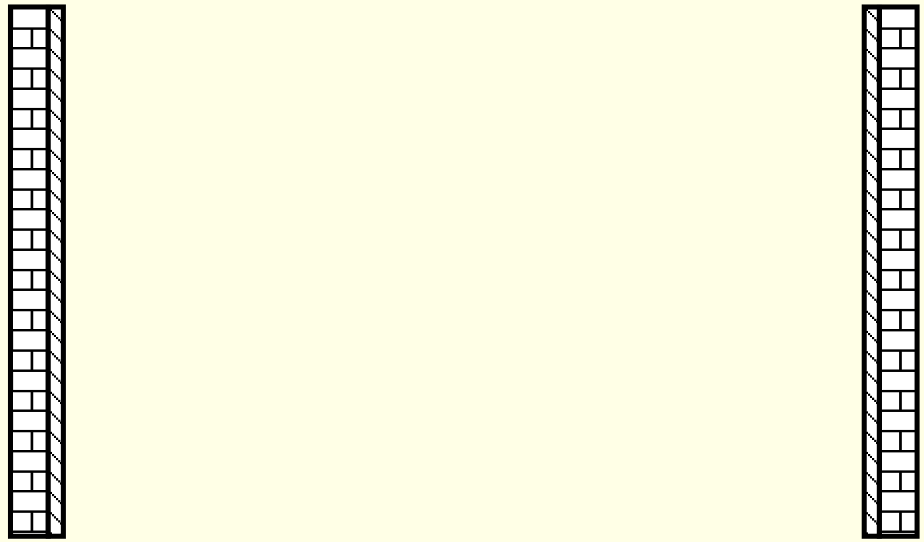


Dependence on defect degree of freedom drops out

Derivation of Rafaels constraint: two boundary sine-Gordon model

Two boundary sine-Gordon model with parameters:

$R(\eta_-, \Theta_-, \gamma_-)$ $\qquad$ $R(\eta_+, \Theta_+, \gamma_+)$

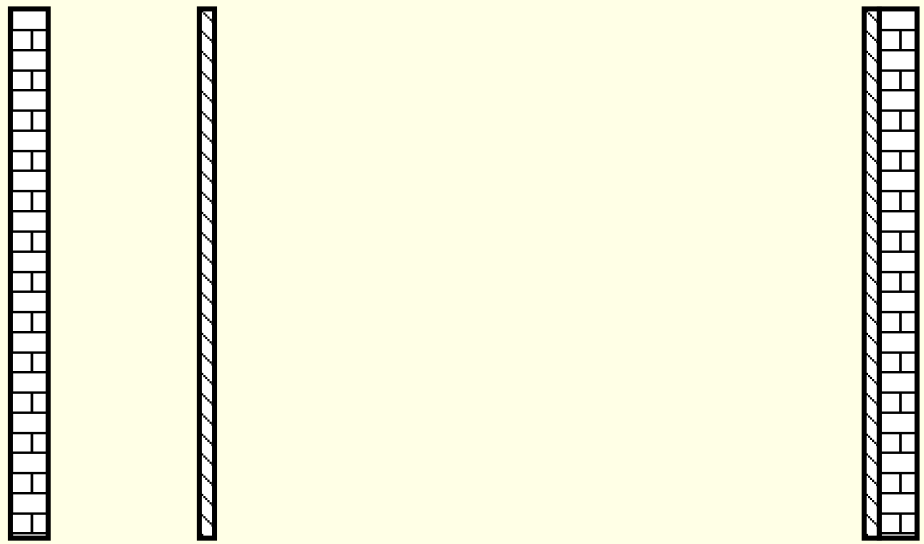


Remove the left defect

Derivation of Rafaels constraint: two boundary sine-Gordon model

Left defect removed

$R_D(\eta_-) \quad T(\Theta_-, \gamma_- - \eta_-, 0) \qquad\qquad R(\eta_+, \Theta_+, \gamma_+)$

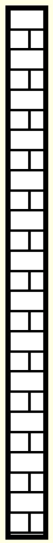


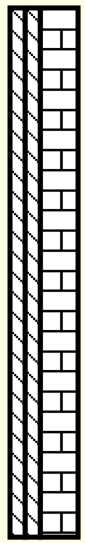
Push the defect to the right

Derivation of Rafaels constraint: two boundary sine-Gordon model

Left defect is pushed to the right boundary

$R_D(\eta_-)$ $\qquad$ $T(\Theta_-,\gamma_- - \eta_-,0)R(\eta_+,\Theta_+,\gamma_+)$





Diagonal right boundary reflection if

$$\boxed{\Theta_- + \Theta_+ = 0 = \eta_- - \eta_+ + \gamma_- - \gamma_+ + \pi(\lambda + 1)}$$

Conclusions on two boundary sine-Gordon model

The system satisfying the constraint is equivalent to a system with diagonal boundary conditions

No need to analyze these nondiagonal cases separately

Two boundary sine-Gordon model without the constraint is essentially different

Consequences for XXZ spin chain

Defect lines show the isospectrality of a double row transfer matrix with diagonal boundary conditions to a nondiagonal one but with the constraint

These cases can be analyzed in the same footing

In the absence of the constraint the system behaves essentially differently

Conclusion for defect theories

Defects are boundary theories (results from boundary apply directly to them)

Defects are simpler then boundary theories (YBE)

($STT = TTS$ instead of $SRSR = RSRS$)

Defects are useful tools (Casimir effect)

Defects help in constructing/classifying boundary R matrices (dressing)

Defects deserve studying